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Math 118 - Spring 2024 - Common Final Exam, version A

Print name:____

Section number:__________ **Instructor's name:**______________________________

Directions:

- This exam has 13 questions. Please check that your exam is complete, but otherwise keep this page closed until the start of the exam is called.
- Fill in your name, and your instructor's name.
- It will be graded out of 100 points.
- Show your work. Answers (even correct ones) without the corresponding work will receive no credit.
- A formula sheet has been provided with this exam. You may not refer to any other notes during the exam.
- You may use a calculator which does not allow internet access. The use of any notes or electronic devices other than a calculator is prohibited.
- **Unless otherwise stated, round any constants to two decimal places if necessary.**

Good luck!

Question:	1	2	3	4	5	6	7
Points:	9	6	12	7	8	6	9
Score:							

Question:	8	9	10	11	12	13	Total
Points:	9	6	12	5	6	5	100
Score:							

1. (9 points) The number of students who took a math class in January 2019 was $1,000$. The number of students taking a math class in January 2024 is 2000.

   (a) Assume that the growth is linear. Find a formula for the function, $S(t)$, the number of students taking a math class t years after 2019.

   (b) Assume that the growth is exponential. Find a formula for the function, $S(t)$, the number of students taking a math class t years after 2019.

   (c) Assume that the growth is exponential. Find the year that there will be 3000 students. Round to the nearest whole number.

2. (6 points) Madeline opens a tutoring business for Math 118 students. Her profits are increasing by an annual growth factor of 1.32. Find the doubling time of the profits for Madeline's tutoring business. That is, find the amount of time it takes for the profits to double.

3. (12 points) Ashley opens a bank account with an initial deposit of $\$10,000$. It earns interest at a nominal rate of 4% per year.

   (a) Find the balance of their account after 3 years if interest is compounded as follows.

   (i) Annually (once per year).

   (ii) Daily (365 times per year).

   (iii) Continuously.

   (b) Ashley starts to think about retirement, and begins to think about their long term savings. She has a choice of the same three accounts from part (a) of this problem, but plans to leave the money in the account for 10 years, instead of 3 years. Which bank account should Ashley choose to maximize their savings? State your reason why in a sentence. Hint: You do not need to do any additional calculations to answer this question

4. (7 points) Consider the exponential function $Q = 42e^{-0.182t}$.

(a) Determine if this function displays exponential growth or decay.
Circle one: **exponential growth** or **exponential decay**. Explain your answer in a sentence.

(b) Give the initial value for this function.

(c) Give the continuous growth rate for this function. Write your answer as a percentage.

(d) Give the domain for the given function.

(e) Give the range for the given function.

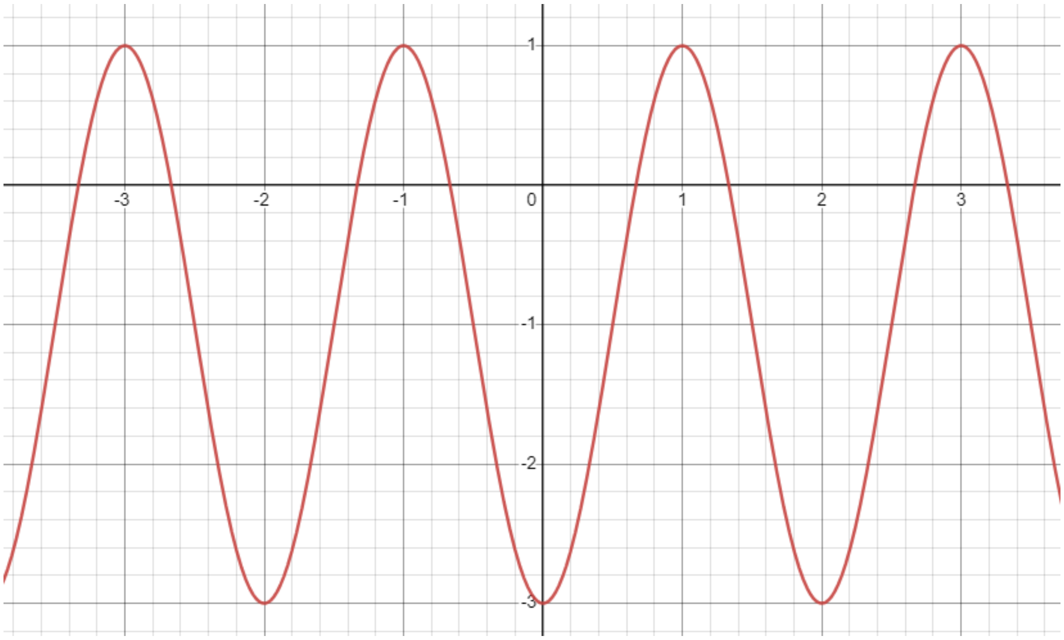
(f) Write the given function in the form $Q = ab^t$, where $b > 0$.

5. (8 points) On January 9th, 2024 low tide at Navy Pier was at midnight. Assume the first high tide of the day is at exactly 6 am. The water level at low tide was 1.8 feet. At high tide, the water was 10.2 feet.

   (a) Use a sinusoidal function $y = A\sin(Bt) + k$ or $y = A\cos(Bt) + k$ to model the height of the water level at Navy pier as a function of time, t, the number of hours since midnight on January 9th.

   (b) Write an equation for the first time the tide is 5 feet high after midnight. Find a solution to this equation, giving your answer in terms of an inverse trig function and also evaluate it with correct units.

6. (6 points) Find a formula of the trigonometric function shown in the graph below.



7. (9 points) For an angle α where $\frac{\pi}{2} < \alpha < \pi$ such that $\cos(\alpha) = -\frac{3}{5}$, find the given quantities without finding α. Give an exact answer for each part.

(a) $\sin(\alpha)$

(b) $\tan(\alpha)$

(c) $\sin(\alpha - \frac{\pi}{3})$

8. (9 points) The top of a 300 foot tower is to be anchored by cables that make an angle of 40° with the ground.

   (a) Draw a picture of the situation

   (b) How long must the cables be?

   (c) How far from the base of the tower should the anchors be placed?

9. (6 points) Let $f(x) = 3x + 4$, $g(x) = 5x - 12$ and $h(x) = e^x$. Find the following, and simplify your answers completely:

   (a) $f(g(2))$

   (b) $h(g(f(x)))$

10. (12 points) Let $P = f(t) = 200(0.82)^t$ be the number of students at Loyola that live in the dorms. Let t be measured in years since 2023.

   (a) Evaluate $f(6)$. Round to the nearest whole number. Describe in words what this quantity represents. Write your answer in a complete sentence with units.

   (b) Find a formula for $f^{-1}(P)$ in terms of P. Give an exact answer.

   (c) Evaluate $f^{-1}(100)$. Round to the nearest whole number

   (d) Describe in words what the quantity you found in part c) represents. Write your answer in a complete sentence with units.

11. (5 points) Decompose the function

$$f(x) = \frac{5}{\sqrt{3x+1}}$$

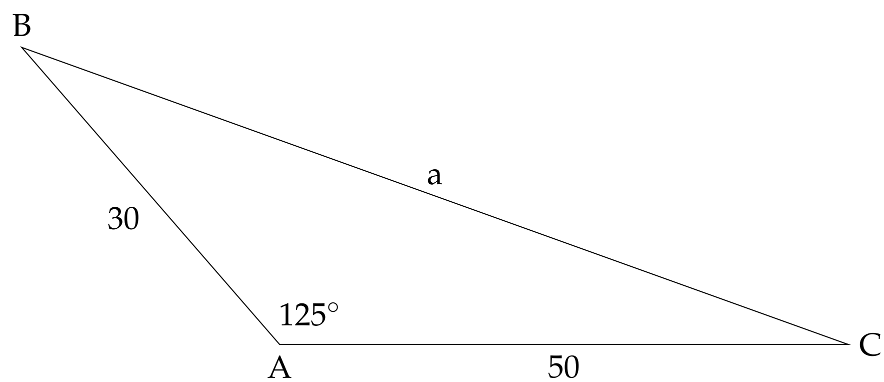
into a composition of two new functions u and v, where v is the inside function. That is $f(x) = u(v(x))$, so that $u(x) \neq x$ and $v(x) \neq x$.

12. (6 points) Perform the following conversions.

    (a) Convert the Cartesian coordinates $(x, y) = (1, \sqrt{3})$ to polar coordinates. Give an exact answer.

    (b) Convert the polar coordinates $(r, \theta) = (4, \frac{\pi}{2})$ to Cartesian coordinates. Give an exact answer.

13. (5 points) Oli and Frank start to run away from point A in different directions. The angle between their two paths is 125 degrees. Oli ran 30 meters from point A to point B. Frank ran 50 meters from point A to point C. How far apart are Oli and Frank? A diagram is below



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Exponential and Logarithm Formulas
Linear Function: $Q(t) = mt + b$
Exponential Function: $Q(t) = a \cdot b^t$
Continuous Exponential Function: $Q(t) = a \cdot e^{kt}$
Simple Interest: $B = P(1 + r)^t$
Compound Interest: $B = P\left(1 + \frac{r}{n}\right)^{nt}$

Trigonometry Formulas
1 radian $= \frac{180}{\pi}$ degrees and 1 degree $= \frac{\pi}{180}$ radians

$$\sin(\theta) = \frac{opp}{hyp} = \frac{y}{r} \quad \cos(\theta) = \frac{adj}{hyp} = \frac{x}{r} \quad \tan(\theta) = \frac{opp}{adj} = \frac{y}{x} = \frac{\sin(\theta)}{\cos(\theta)}$$

$$\csc(\theta) = \frac{1}{\sin(\theta)} = \frac{r}{y} \quad \sec(\theta) = \frac{1}{\cos(\theta)} = \frac{r}{x} \quad \cot(\theta) = \frac{1}{\tan(\theta)} = \frac{x}{y} = \frac{\cos(\theta)}{\sin(\theta)}$$

Pythagorean Identities: $\sin^2(\theta) + \cos^2(\theta) = 1 \quad \tan^2(\theta) + 1 = \sec^2(\theta) \quad 1 + \cot^2(\theta) = \csc^2(\theta)$

Sum and Difference Formulas:
$\sin(A + B) = \sin(A)\cos(B) + \cos(A)\sin(B)$
$\sin(A - B) = \sin(A)\cos(B) - \cos(A)\sin(B)$
$\cos(A + B) = \cos(A)\cos(B) - \sin(A)\sin(B)$
$\cos(A - B) = \cos(A)\cos(B) + \sin(A)\sin(B)$

Even-Odd Identities: $\sin(-x) = -\sin(x)$ and $\cos(-x) = \cos(x)$ and $\tan(-x) = -\tan(x)$

Other identities: $\sin(\theta) = \sin(\pi - \theta)$, $\cos(\theta) = -\cos(\pi - \theta)$ and $\tan(\theta) = -\tan(\pi - \theta)$

General form for sine and cosine: $f(t) = A\sin(Bt) + k$ and $f(t) = A\cos(Bt) + k$

General form with horizontal shift: $f(t) = A\sin(B(t - h)) + k$ and $f(t) = A\cos(B(t - h)) + k)$

Period for sine and cosine: $P = \frac{2\pi}{|B|}$ or $PB = 2\pi$. Amplitude $= |A| = \frac{max - min}{2}$. Midline: $y = k$, where $k = \frac{max + min}{2}$

Law of Sines: $\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$

Law of Cosines: $c^2 = a^2 + b^2 - 2ab\cos(C)$

Arc Length: $s = r\theta$

Inverse Trig Functions
$\theta = \cos^{-1}(y)$ provided that $y = \cos(\theta)$ and $0 \leq \theta \leq \pi$
$\theta = \sin^{-1}(y)$ provided that $y = \sin(\theta)$ and $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$
$\theta = \tan^{-1}(y)$ provided that $y = \tan(\theta)$ and $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$
Polar coordinates conversions
$r^2 = x^2 + y^2$, $\tan(\theta) = \frac{y}{x}$, $x = r\cos(\theta)$, $y = r\sin(\theta)$

The Unit Circle